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4.1 EXTREME VALUES OF

Derivatives can tell us if we have maximum or minimum values, and if so, where they exist.

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Def^n

Let f be a function with domain, D. Then f(c) is the:

(a) Absolute minimum value on D, iff $f(x) \geq f(c)$ for all *x* in D.

(b) Absolute maximum value on D, iff $f(x) \leq f(c)$ for all *x* in D.

(c) Absolute or global max and min values are also called absolute extrema.

Exploration

Explore $y=x^2$ on the following domains:

$(-\infty,\infty)$

$[0,2]$

$(0,2]$

$(0,2)$

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Extreme Value Theorem

If f is continuous on a closed interval [a,b] then f has both a maximum value and a minimum value on the interval.

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Local (Relative) Extreme Values

Let c be an interior point in the domain of the function f. Then f(c) is a:

Local minimum value at c, iff $f(x) \geq f(c)$ for all x in an open interval containing c.

Local maximum value on D, iff $f(x) \leq f(c)$ for all x in an open interval containing c.

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Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c on its domain, and if f ` exists at c, then

$$f`(c)=0$$

Critical Point

A point in the interior of the domain of a function f at which f `=0 or f ` does not exist is a critical point of f.

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Ex. 4

Find the extreme values of

$$f(x)=\begin{cases}2-3x^2 & x\le 1\\ 2x-3 & x>1\end{cases}$$

Ex. 5

Using graphical methods, find all extreme values of:

$$f(x)=\ln\left|\frac{x}{1+x^2}\right|$$

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Rolle's Theorem

If f(a)=f(b)=0, then there is at least one point where f'(c)=0. ONLY for polynomials.